Trigonometric Identities

The Six Trigonometric Functions

$$\sin\theta = \frac{opp}{hyp} = \frac{y}{r} \qquad \csc\theta = \frac{hyp}{opp} = \frac{r}{y}$$
$$\cos\theta = \frac{adj}{hyp} = \frac{x}{r} \qquad \sec\theta = \frac{hyp}{adj} = \frac{r}{x}$$
$$\tan\theta = \frac{opp}{adj} = \frac{y}{x} \qquad \cot\theta = \frac{adj}{opp} = \frac{x}{y}$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$
$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Sum or Difference of Two Angles

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\cos\alpha\cos\beta = \tfrac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$$
$$\cos\alpha\sin\beta = \tfrac{1}{2}(\sin(\alpha+\beta)-\sin(\alpha-\beta))$$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \qquad \cos 2\theta = 2\cos^2\theta - 1$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$

Sum to Product Formulas

$$\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha\pm\beta}{2}\right)\cos\left(\frac{\alpha\mp\beta}{2}\right)$$
$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Half-Angle Formulas

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\frac{\theta}{2} = \csc\theta - cot\theta \qquad \cot\frac{\theta}{2} = \csc\theta + cot\theta$$
$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$$
$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta}$$
$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$$

Co-Function Identities

$$\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos\theta \qquad \csc\left(\tfrac{\pi}{2}-\theta\right) = \sec\theta$$
$$\cos\left(\tfrac{\pi}{2}-\theta\right) = \sin\theta \qquad \sec\left(\tfrac{\pi}{2}-\theta\right) = \csc\theta$$
$$\tan\left(\tfrac{\pi}{2}-\theta\right) = \cot\theta \qquad \cot\left(\tfrac{\pi}{2}-\theta\right) = \tan\theta$$

Even-Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$
$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$
$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Complex Numbers

$$e^{i\pi} = \cos\theta + i\sin\theta \qquad \cos\theta = \tfrac{1}{2}\left(e^{i\theta} + e^{-i\theta}\right)$$
$$e^{i\theta} = \cos\theta + i\sin\theta \qquad \sin\theta = \tfrac{1}{2i}\left(e^{i\theta} - e^{-i\theta}\right)$$
$$e^{-i\theta} = i\cos\theta - i\sin\theta \qquad \tan\theta = \frac{e^{i\theta} - e^{-i\theta}}{i\left(e^{i\theta} + e^{-i\theta}\right)}$$

Expansions

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \csc x = \frac{1}{x} + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \cdots$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad \sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \cdots$$
$$\tan x = x - \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \cdots \qquad \cot x = \frac{1}{x} - \frac{x}{3} - \frac{1x^3}{45} - \frac{2x^5}{945} - \cdots$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

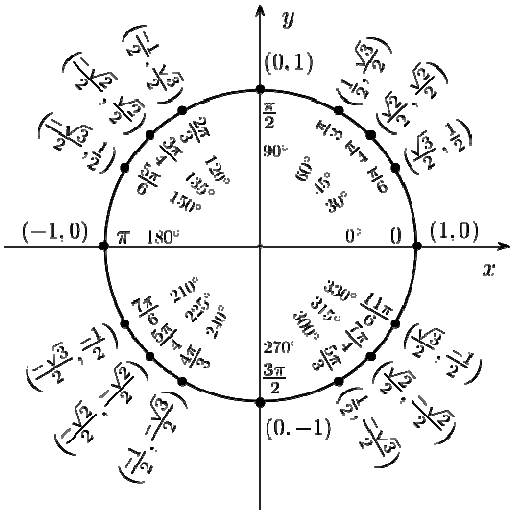
Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos C \quad \text{or} \quad C = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right)$$

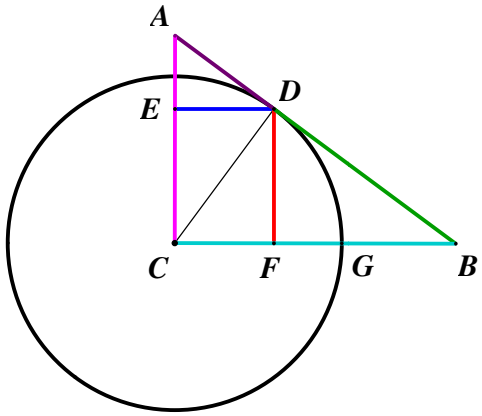
Each trigonometric function in terms of the others

In terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\dfrac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\dfrac{1}{\csc\theta}$	$\pm\dfrac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\dfrac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\dfrac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\dfrac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\dfrac{1}{\sec\theta}$	$\pm\dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\dfrac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\dfrac{1}{\cot\theta}$
$\csc\theta =$	$\dfrac{1}{\sin\theta}$	$\pm\dfrac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\dfrac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\dfrac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\dfrac{1}{\sqrt{1-\sin^2\theta}}$	$\dfrac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\dfrac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\dfrac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\dfrac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\dfrac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\dfrac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Unit Circle



Geometric Constructions of Trig Functions



Circle with radius $\overline{CD}$

$\overline{DF}$ – sine $\overline{DE}$ – cosine $\overline{DB}$ – tangent

$\overline{CA}$ – cosecant $\overline{CB}$ – secant $\overline{DA}$ – cotangent

Trigonometric Values for Common Angles

Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°	0π	0	1	0	undefined	1	undefined
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$	2	$2\sqrt{3}/3$	$\sqrt{3}$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$2\sqrt{3}/3$	2	$\sqrt{3}/3$
90°	$\pi/2$	1	0	undefined	0	undefined	1
120°	$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$	$2\sqrt{3}/3$	-2	$-\sqrt{3}/3$
135°	$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°	$5\pi/6$	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$	2	$-2\sqrt{3}/3$	$-\sqrt{3}$
180°	π	0	-1	0	undefined	-1	undefined
210°	$7\pi/6$	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}/3$	-2	$-2\sqrt{3}/3$	$\sqrt{3}$
225°	$5\pi/4$	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°	$4\pi/3$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$	$-2\sqrt{3}/3$	-2	$\sqrt{3}/3$
270°	$\pi/2$	-1	0	undefined	0	undefined	-1
300°	$5\pi/3$	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$	$-2\sqrt{3}/3$	2	$-\sqrt{3}/3$
315°	$7\pi/4$	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330°	$11\pi/6$	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}/3$	-2	$2\sqrt{3}/3$	$-\sqrt{3}$
360°	2π	0	1	0	undefined	1	undefined